كلية الهندســـة بشبرا
FACULTY OF ENGINEERING- SHOUBRA

Electromagnetic Fields

Sheet (04)

Solved Problems

Assoc. Prof. Dr/ Moataz M. Elsherbini

motaz.ali@feng.bu.edu.eg

EXAMPLE 1

Example (1)

Considering electromagnetic waves, in free space, or sourceless (ρ_v = 0, J = 0 and σ = 0). If this electric field intensity, $\vec{E}$ is given by:

$$\vec{E}(z,t) = 0.5\ \cos(2\pi x 10^9 t - 20.9z)\,\hat{a}_x\ V/m$$

Determine the following:

a) The phasor representation of the electric field, $\vec{E}$.

b) The magnetic field intensity, $\vec{H}$ associated with the wave in both phasor and time domain representations.

c) The intrinsic impedance of the free space, η, defined as $\eta = \dfrac{|\vec{E}_s|}{|\vec{H}_s|}[\Omega]$

Solution:

a) Step #1: Write the exponential notation or equation as:

$$\vec{E}(z,t) = \mathrm{Re}[0.5\ e^{j(2\pi x 10^9 t - 20.9z)}]\hat{a}_x\ V/m$$

Step #2: drop Re and suppress $e^{j2\pi 10^9 t}$ to obtaining the phasor:

$$\vec{E}_s(z) = 0.5\ e^{-j20.9z}\hat{a}_x\ V/m$$

b) Apply the Maxwell's equation for time-varying Harmonic fields given in Table 4.2 as:

$$\nabla x\vec{E}_s = -j\omega\vec{B}_s \ \Rightarrow \nabla x\vec{E}_s = -j\mu\omega\vec{H}_s$$

4.5 Time Harmonic Fields (Continued)

Example 4.6 Solution: In matrix form, we have:

$$Curl\vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0.5\ e^{-j20.9z} & 0 & 0 \end{vmatrix} = 0\hat{a}_x - 0.5(j20.9)e^{-j20.9z}\hat{a}_y + 0\hat{a}_z$$

$$\nabla x\vec{E}_s = -j\omega\vec{B}_s \;\Rightarrow\; 0\hat{a}_x - 0.5(j20.9)e^{-j20.9z}\hat{a}_y + 0\hat{a}_z = -j\omega\mu(H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z$$

$$j\omega\mu H_y = 0.5(j20.9)e^{-j20.9z} \;\Rightarrow\; H_y = \frac{10.47}{\omega\mu}e^{-j20.9z} \quad H_x = 0 \ \ and\ H_z = 0$$

The magnetic field, $\vec{H}$ in phasor is given by: $\vec{H}_s(z) = \dfrac{10.47}{\omega\mu}e^{-j20.9z}\hat{a}_y\ \ A/m$

From the equation, the frequency f = 10^9 Hz = 1 GHz and μ = μ_o = $4\pi x10^{-7}$, so:

$$\vec{H}_s(z) = \frac{10.47}{2\pi x10^9\, x4\pi x10^{-7}}e^{-j20.9z}\hat{a}_y = 1.33e^{-j20.9z}\hat{a}_y\ \ mA/m$$

$\vec{H}$ in time domain is: $\vec{H}(z,t) = 1.33\cos(2\pi x10^9 t - 20.9z)\hat{a}_y\ \ mA/m$

c) The intrinsic impedance $\eta = \dfrac{|\vec{E}_s|}{|\vec{H}_s|} = \dfrac{0.5}{1.33x10^{-3}} = 377\ \ or\ \ 12\pi\ \ [\Omega]$

EXAMPLE 2

The conduction current flowing through a wire with conductivity, $\sigma = 3 \text{ X } 10^7$ S/m and relative permittivity $\varepsilon_r = 1$ is given by $I_c = 3 \sin \omega t$ [mA]. If $\omega = 10^8$ rad/s, find the displacement current, I_d.

Solution: The displacement current density, $\vec{J}_d$ is given by (4.18b) as:

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

Since the conduction current, I_c is given, the electric field intensity $\vec{E}$ is found from conduction current density (4.11) as:

$$\vec{J}_{cond} = \sigma \vec{E} \qquad \Rightarrow I_c = \sigma |\vec{E}| A = \sigma E A$$

$$E = \frac{I_c}{\sigma A} = \frac{3x10^{-3} \sin \omega t}{3x10^7 A} = \frac{1x10^{-10}}{A} \sin \omega t \ [V/m] \Rightarrow \frac{\partial \vec{E}}{\partial t} = \left(\frac{1x10^{-10}}{A} \right) \omega \cos \omega t$$

$$\vec{J}_d = \varepsilon \omega \left(\frac{1x10^{-10}}{A} \right) \cos \omega t = 8.85x10^{-12} x10^8 \left(\frac{1x10^{-10}}{A} \right) \cos \omega t = \frac{8.85x10^{-14}}{A} \cos \omega t$$

$$I_d = J_d x A = 8.85x10^{-14} \cos \omega t \ [A]$$

EXAMPLE 3

If a wave has $\bar{B} = 4x\hat{a_x}$ is This wave Satisfy
$\bar{E} = 2\hat{a_y}$ Maxwell's equ?

Sol/ we apply 4 equations of Maxwell's & if one of them Failed so the wave not satisfy Maxwell eq.

here if we apply $\nabla \cdot \bar{B} = 0 \longrightarrow$ Fail

$\nabla \cdot \bar{B} = 4 = \frac{\partial}{\partial x} \bar{B}_x \hat{a}_x + \frac{\partial}{\partial y} \bar{B}_y \hat{a}_y + \frac{\partial}{\partial z} \bar{B}_z \hat{a}_z$

$= \boxed{4}$

Because

EXAMPLE 4

A poor conductor is characterized by a conductivity, σ = 100 S/m (s/m) and relative permittivity, ε_r = 4. At what angular frequency, ω the amplitude of the conduction current density J_c equal to the amplitude of the displacement density, J_d.

Solution: The displacement current density, $\vec{J}_d$ is given by (4.18b) as:

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t} \qquad \text{and} \qquad \vec{J}_{cond} = \sigma \vec{E}$$

Consider amplitudes only, we get:

$$|\vec{J}_{cond}| = |\vec{J}_d| \quad \Rightarrow \sigma E = \varepsilon \omega E \quad \Rightarrow \sigma = \varepsilon \omega$$

Therefore: $$100 = \omega x 4 x 8.85 x 10^{-12}$$

and

$$\omega = \frac{100}{4x8.85x4x8.85x10^{-12}} = 2.82x4x8.85x10^{12} \ rad/s$$

EXAMPLE 5

repeat problem (7) for

$\overline{H} = \frac{1}{\pi} \cos(3\times10^8 t - 4x)\hat{a_z}$

$\overline{H} = \frac{1}{\pi} \cos(3\times10^8 t - 4x)\hat{a_y}$

$E = 60 \cos(3\times10^8 t -$

$\boxed{\begin{array}{l} \sigma = 0 \\ \mu_r = 2 \\ \epsilon_r = 8 \end{array}}$

$\nabla\cdot\overline{B} = 0$

$\nabla\cdot\overline{D} = 0$

$\nabla\times\overline{E} = -\frac{\partial\overline{B}}{\partial t}$

$\nabla\times\overline{H} = J + \frac{\partial D}{\partial t}$

Sol $\nabla\cdot\overline{B} = 0$

$\frac{\partial}{\partial x}\overline{B_x}\hat{a_x} + \frac{\partial}{\partial y}\overline{B_y}\hat{a_y} + \frac{\partial \hat{B_z}}{\partial z}\hat{a_z}$

B_z f of $\cancel{x}$ (not z)

$B_x = B_y = 0$ $\quad \therefore \frac{\partial B_z}{\partial z} = 0$

(1)

$\therefore \nabla\cdot\overline{B} = 0$ شرط الحقيق

$\nabla\cdot\overline{D} = \nabla\cdot\epsilon\overline{E}$

E_y f of $\cancel{x}$ while

$E_x, E_z = 0$

$\therefore \nabla\cdot E = 0$ Because $E_x = 0$ (2)

$E_z = 0$

$\frac{\partial E_y}{\partial y} = 0$ (f of x)

: حققت معادلتين لماكسويل

$$\boxed{\nabla \times \bar{E} = -\frac{\partial B}{\partial t}}$$

$$\mu_0 \mu_r = 4\pi \times 10^{-7} \times 2$$

$$-\frac{\partial B}{\partial t} = -\frac{\partial \mu \bar{H}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$= +8\pi \times 10^{-7} * \frac{3\times 10^{8}}{\pi} \sin(3\times 10^{8} t - 4x)$$

$$\bar{H} = \frac{1}{\pi} \cos(3\times 10^{8} t - 4x)\, a_y$$

$$\frac{\partial H}{\partial t} = -\frac{3\times 10^{8}}{\pi} \sin(3\times 10^{8} t - 4x)$$

$$\therefore \boxed{-\frac{\partial B}{\partial t} = 240 \sin(3\times 10^{8} t - 4x)}$$

Now

$$\nabla \times \bar{E} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = +\frac{\partial E_y}{\partial x} \hat{a_z} \quad \cancel{-\frac{\partial E_y}{\partial z} \hat{a_x}} \rightarrow 0$$

لا يوجد داخلها z

$$\therefore \nabla \times \bar{E} = 240 \sin(3\times 10^{8} - 4x)$$

(3)

تحققت، تساوي ؟

$$\boxed{\bar{\nabla} \times \bar{H} = \frac{\partial \bar{D}}{\partial t}}$$

$$\frac{\partial \bar{D}}{\partial t} = \frac{\partial \epsilon \bar{E}}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t} = -8.85 \times 10^{-12} \times \underset{\epsilon_r}{8} \times \underset{\omega}{3 \times 10^{8}} \sin(\longleftarrow)$$

$$= -\frac{4}{\pi} \sin(3 \times 10^{8} t - 4x)\,\hat{a}_y$$

$$\nabla \times \bar{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix} = -\frac{\partial H_z}{\partial x}\,\hat{a}_y$$

$$= -\hat{a}_y \frac{4}{\pi} \sin(3 \times 10^{8} t - 4x)$$

$$\because \sigma = 0 \;\; , \; J_c = 0 = \sigma E$$

$$, \; \nabla \times \bar{H} = J_c + \frac{\partial \bar{D}}{\partial t}$$

EXAMPLE 6

(a) For series R_C Circuit connected to AC source of $V = V_m \sin\omega t$, Prove that the displacement current of capacitor = conduction current.

Current I_c where $V = V_m \sin \omega t$

Solution

The conduction current for a capacitor $I_c = C \dfrac{dV}{dt}$

$$= C V_m \omega \cos \omega t$$

$$I_d = \iint_A \frac{\partial \bar{D}}{\partial t} \cdot d\bar{s} = \epsilon \frac{\partial}{\partial t} \iint_A \bar{E} \cdot d\bar{A}$$

$$I_d = \iint_A \frac{\partial \bar{D}}{\partial t} \cdot d\bar{s} = \epsilon \frac{\partial}{\partial t} \iint_A \bar{E} \cdot d\bar{A}$$

$$= \frac{\epsilon A}{d} \frac{\partial V}{\partial t} = C V_m \omega \cos \omega t$$

$$\therefore \boxed{I_c = I_d}$$

EXAMPLE 7

A circular cross-section conductor of radius, r = 2 mm carries an ac current, ic of ic = 2.5 sin (5×10^8 t) μA. What is the amplitude of the displacement current density if the conductivity of the materials, σ= 35 MS/m and the relative permittivity, εr= 1?

The ratio of conduction to displacement current density is

$$J_c = \sigma E \qquad J_D = \varepsilon \frac{\partial E_s}{\partial t}$$

$$\text{if} \quad E_s = E \sin \omega t \qquad \frac{\partial E_s}{\partial t} = E\omega \cos \omega t.$$

Taking only the amplitude, $\varepsilon \frac{\partial E_s}{\partial t} = \omega \varepsilon E$

$$\frac{J_c}{J_D} = \frac{\sigma E}{\omega \varepsilon E} = \frac{\sigma}{\omega \varepsilon} = \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} = \frac{35 \times 10^6}{(5 \times 10^8) \cdot 8.85 \times 10^{-12}} \quad [\because \ \varepsilon_r = 1]$$

$$= 79.09 \times 10^8.$$

We know $J_c = I_c / \text{Area}$ $\qquad$ $\text{Area} = \pi a^2 = \pi \times (2 \times 10^{-3})^2$

$$J_c = \frac{2.5 \times 10^{-6}}{\pi \times (2 \times 10^{-3})^2} = 0.198$$

$$J_D = \frac{J_c}{79.09 \times 10^8} = \frac{0.198}{79.09 \times 10^8} = 2.515 \times 10^{-11} \text{ A/m}^2.$$

EXAMPLE 8

a) Express the vector $\vec{A}_1(z,t) = 10\cos(10^8 t - 10z + 60^o)\hat{a}_x$ in phasor form.

b) Express the vector $\vec{B}_s(x,y) = e^{jx}(\hat{a}_x - \hat{a}_z)\sin(\pi y)$ in instantaneous form.

c) Express the vector $\vec{A}_2(x,t) = 2\sin(10t + x - \pi/4)\hat{a}_y$ in phasor form.

a) $\vec{A}_{1s}(z) = 10e^{j(60^o - 10x)}\hat{a}_y$

b) $\vec{B}(x,y,t) = \sin(\pi y)\cos(\omega t + x)(\hat{a}_x - \hat{a}_z)$

c) $\vec{A}_{2s}(x) = 2e^{j(x - 3\pi/4)}\hat{a}_y$